

## A paradox on quantum field theory of neutrino mixing and oscillations

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ABSTRACT: Neutrino mixing and oscillations in quantum field theory framework had been studied before, which show that the Fock space of flavor states is unitarily inequivalent to that of mass states (*inequivalent vacua model*). A paradox emerges when we use these neutrino weak states to calculate the amplitude of  $W$  boson decay. The branching ratio of  $W^+ \rightarrow e^+ + \nu_\mu$  to  $W^+ \rightarrow e^+ + \nu_e$  is approximately at the order of  $O(m_i^2/k^2)$ . This existence of flavor changing currents contradicts to the Hamiltonian we started from, and the usual knowledge about weak processes. Also, negative energy neutrinos (or violating the principle of energy conservation) appear in this framework. We discuss possible reasons for the appearance of this paradox.

KEYWORDS: Neutrino Physics, Solar and Atmospheric Neutrinos.

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**1. Introduction**

Neutrino oscillation experiments[1–6] give compelling evidences for neutrino oscillation theory [7–11]. But there are some difficulties in theoretical aspects about the mixing fields in Quantum Field Theory (QFT), such as the definition of weak states[12, 13], or equivalently the definitions of the operators for creating and annihilating a weak state particle.

The *inequivalent vacua model*[14–19] is constructed with a preceding attitude. In this model the transformation between Fock space of mass states and flavor states is a *Bogliubov* transformation. Basic results of this model are: unitary in-equivalence between mass vacuum and flavor vacuum; fermion condensation in vacuum responsible for correction to the usual oscillation formulas and so on. An exact neutrino oscillation formula is obtained there, which leads the usual Pontecovo’s oscillation formula to an approximate convenience. In this model, there is freedom to choose spinors to expand the flavor fields  $\nu_\sigma(x)$ . We can use a series of spinors  $\{u_\sigma(\mathbf{k}, r), v_\sigma(\mathbf{k}, r)\}$ [20, 21], which satisfy free Dirac equations

$$(\not{k} - \mu_\sigma)u_\sigma(\mathbf{k}, r) = 0, \tag{1.1}$$

$$(\not{k} + \mu_\sigma)v_\sigma(\mathbf{k}, r) = 0, \tag{1.2}$$

where  $\mu_\sigma$  are free mass parameters. This degree of freedom implies that we have infinite equivalent Fock space. In respect that, the author in ref.[22] thinks that the arbitrary parameters  $\mu_\sigma$  can be physical observables, so he argues that Fock states of flavor neutrinos are unphysical [22, 23]. But the authors in ref.[17, 19] demonstrate that the oscillation formulas in vacuum are free from the arbitrariness of the mass parameter  $\mu_\sigma$ . So we will omit this problem in this paper, and use their initial expansions. Our focus is to study weak processes in this *inequivalent vacua model*. The results come out that a paradox appears even if we carry out everything correctly.

The paper is organized as follows: in section 2, we give the basic aspects of the *inequivalent vacua model*; in section 3, we will derive our calculations for  $W$  boson decay and give our main results of this paper; in section 4, we give the conclusions and comments.

## 2. Basic aspects of the *inequivalent vacua model*

Following the previous study of the neutrino mixing in QFT [14–21], In this section we start our derivations in a two-generation case, and will give general formulas for  $N$  generations at the end of this section, which are useful in our main calculations in this paper. The *Bogliubov* transformation is defined as

$$\begin{aligned} \begin{pmatrix} \nu_e(x) \\ \nu_\mu(x) \end{pmatrix} &= G^{-1}(\theta; t) \begin{pmatrix} \nu_1(x) \\ \nu_2(x) \end{pmatrix} G(\theta; t) \\ &= \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} \nu_1(x) \\ \nu_2(x) \end{pmatrix}. \end{aligned} \quad (2.1)$$

$G(\theta; t)$  is given by

$$G(\theta; t) = \exp\left\{\theta \int d^3\mathbf{x} [\nu_1^\dagger(x)\nu_2(x) - \nu_2^\dagger(x)\nu_1(x)]\right\}, \quad (2.2)$$

where  $t = x_0$ ,  $\{\nu_\sigma(x), \sigma = e, \mu\}$  and  $\{\nu_i(x), i = 1, 2\}$  are the neutrino fields with definite flavors and masses, respectively.

The mass fields are expanded as

$$\begin{aligned} \nu_i(x) &= \frac{1}{(2\pi)^{3/2}} \sum_r \int d^3\mathbf{k} [u_i(\mathbf{k}, r) a_{\mathbf{k},i}^r e^{-i\omega_i t} + v_i(-\mathbf{k}, r) b_{-\mathbf{k},i}^{r\dagger} e^{i\omega_i t}] e^{i\mathbf{k}\cdot\mathbf{x}} \\ &\equiv \frac{1}{(2\pi)^{3/2}} \sum_r \int d^3\mathbf{k} [u_i(\mathbf{k}, r) a_{\mathbf{k},i}^r(t) + v_i(-\mathbf{k}, r) b_{-\mathbf{k},i}^{r\dagger}(t)] e^{i\mathbf{k}\cdot\mathbf{x}}, \end{aligned} \quad (2.3)$$

where  $\omega_i = \sqrt{\mathbf{k}^2 + m_i^2}$ ,  $u_i(\mathbf{k}, r)$  and  $v_i(-\mathbf{k}, r)$  are the solutions of free Dirac equations in momentum space with definite spin  $r$  and mass  $m_i$ :

$$(\not{k} - m_i)u_i(\mathbf{k}, r) = 0, \quad (2.4)$$

$$(\not{k} + m_i)v_i(\mathbf{k}, r) = 0. \quad (2.5)$$

The Hilbert space of definite mass states  $\mathcal{H}_{1,2}$  is constructed by operators  $a_{\mathbf{k},i}^r(t)$  and  $b_{\mathbf{k},i}^{r\dagger}(t)$ . So the mass vacuum  $|0\rangle_m$  is defined as

$$\begin{pmatrix} a_{\mathbf{k},i}^r(t) \\ b_{-\mathbf{k},i}^{r\dagger}(t) \end{pmatrix} |0\rangle_m = 0, \quad (2.6)$$

with normalization  ${}_m\langle 0|0\rangle_m = 1$ .

As discussed above, we will use the initial expansions of flavor fields in ref.[14–16]. The explicit forms are

$$\nu_\sigma(x) = \frac{1}{(2\pi)^{3/2}} \sum_r \int d^3\mathbf{k} [u_i(\mathbf{k}, r) a_{\mathbf{k},\sigma}^r(t) + v_i(-\mathbf{k}, r) b_{-\mathbf{k},\sigma}^{r\dagger}(t)] e^{i\mathbf{k}\cdot\mathbf{x}}, \quad (2.7)$$

where  $(\sigma, i)$  stands for either  $(e, 1)$  or  $(\mu, 2)$ .

Immediately we obtain

$$\begin{pmatrix} a_{\mathbf{k},\sigma}^r(t) \\ b_{-\mathbf{k},\sigma}^{r\dagger}(t) \end{pmatrix} = G^{-1}(\theta; t) \begin{pmatrix} a_{\mathbf{k},i}^r(t) \\ b_{-\mathbf{k},i}^{r\dagger}(t) \end{pmatrix} G(\theta; t). \quad (2.8)$$

The vacuum for flavor states is

$$|0(t)\rangle_f = G^{-1}(\theta; t)|0\rangle_m. \quad (2.9)$$

Note that the vacuum  $|0(t)\rangle_f$  is time-dependent, so do the creation and annihilation operators of flavor states.

The explicit matrix form for flavor operators is

$$\begin{pmatrix} a_{\mathbf{k},e}^r(t) \\ a_{\mathbf{k},\mu}^r(t) \\ b_{-\mathbf{k},e}^{r\dagger}(t) \\ b_{-\mathbf{k},\mu}^{r\dagger}(t) \end{pmatrix} = \begin{pmatrix} c_\theta \rho_{1,1}^{\mathbf{k}} & s_\theta \rho_{1,2}^{\mathbf{k}} & ic_\theta \lambda_{1,1}^{\mathbf{k}} & is_\theta \lambda_{1,2}^{\mathbf{k}} \\ -s_\theta \rho_{2,1}^{\mathbf{k}} & c_\theta \rho_{2,2}^{\mathbf{k}} & -is_\theta \lambda_{2,1}^{\mathbf{k}} & ic_\theta \lambda_{2,2}^{\mathbf{k}} \\ ic_\theta \lambda_{1,1}^{\mathbf{k}} & is_\theta \lambda_{1,2}^{\mathbf{k}} & c_\theta \rho_{1,1}^{\mathbf{k}} & s_\theta \rho_{1,2}^{\mathbf{k}} \\ -is_\theta \lambda_{2,1}^{\mathbf{k}} & ic_\theta \lambda_{2,2}^{\mathbf{k}} & -s_\theta \rho_{2,1}^{\mathbf{k}} & c_\theta \rho_{2,2}^{\mathbf{k}} \end{pmatrix} \begin{pmatrix} a_{\mathbf{k},1}^r(t) \\ a_{\mathbf{k},2}^r(t) \\ b_{-\mathbf{k},1}^{r\dagger}(t) \\ b_{-\mathbf{k},2}^{r\dagger}(t) \end{pmatrix}, \quad (2.10)$$

where  $c_\theta \equiv \cos \theta$ ,  $s_\theta \equiv \sin \theta$  and

$$\rho_{i,j}^{\mathbf{k}} \delta_{rs} \equiv \cos \frac{\chi_i - \chi_j}{2} \delta_{rs} = u_i^\dagger(\mathbf{k}, r) u_j(\mathbf{k}, s) = v_i^\dagger(-\mathbf{k}, r) v_j(-\mathbf{k}, s), \quad (2.11)$$

$$i\lambda_{i,j}^{\mathbf{k}} \delta_{rs} \equiv i \sin \frac{\chi_i - \chi_j}{2} \delta_{rs} = u_i^\dagger(\mathbf{k}, r) v_j(-\mathbf{k}, s) = v_i^\dagger(-\mathbf{k}, r) u_j(\mathbf{k}, s), \quad (2.12)$$

with  $i, j = 1, 2$  and  $\cot \chi_i = |\mathbf{k}|/m_i$ .

For  $N$  generations, general formulas are similar to (2.10):

$$a_{\mathbf{k},\sigma}^r(t) = \sum_{j=1}^N \{U_{\sigma,j} \rho_{i,j}^{\mathbf{k}} a_{\mathbf{k},j}^r(t) + U_{\sigma,j} i\lambda_{i,j}^{\mathbf{k}} b_{-\mathbf{k},j}^{r\dagger}(t)\}, \quad (2.13)$$

$$b_{-\mathbf{k},\sigma}^{r\dagger}(t) = \sum_{j=1}^N \{U_{\sigma,j} i\lambda_{i,j}^{\mathbf{k}} a_{\mathbf{k},j}^r(t) + U_{\sigma,j} \rho_{i,j}^{\mathbf{k}} b_{-\mathbf{k},j}^{r\dagger}(t)\}, \quad (2.14)$$

where the pair of  $(\sigma, i)$  denotes  $((e, 1), (\mu, 2), (\tau, 3), \dots)$ , and  $U_{\sigma,j}$  is the neutrino mixing matrix if we choose the charge leptons to be the mass eigenstates.

We compute the anticommutations of these operators at different time by fixing one operator at  $t = 0$ , and the other at time  $t$ :

$$\{a_{\mathbf{k},\sigma}^r(0), a_{\mathbf{k},\delta}^{r\dagger}(t)\} = \sum_l U_{\sigma,l} U_{\delta,l}^* \{\rho_{i,l}^{\mathbf{k}} \rho_{j,l}^{\mathbf{k}} e^{i\omega_l t} + \lambda_{i,l}^{\mathbf{k}} \lambda_{j,l}^{\mathbf{k}} e^{-i\omega_l t}\}, \quad (2.15)$$

$$\{a_{\mathbf{k},\sigma}^r(0), b_{-\mathbf{k},\delta}^{r\dagger}(t)\} = \sum_l U_{\sigma,l} U_{\delta,l}^* \{-i\rho_{i,l}^{\mathbf{k}} \lambda_{j,l}^{\mathbf{k}} e^{i\omega_l t} + i\lambda_{i,l}^{\mathbf{k}} \rho_{j,l}^{\mathbf{k}} e^{-i\omega_l t}\}, \quad (2.16)$$

where the pairs of  $(\sigma, i)$  and  $(\delta, j)$  denote  $((e, 1), (\mu, 2), (\tau, 3), \dots)$ .

The most distinct property of this model is the time dependence of the flavor neutrino ladder operators and the non-standard canonical anticommutations at different time. The

flavor changing effect in (2.15) and (2.16) gives important results of this paper. In order to give an explicit origin of the flavor changing current effect (see Section 3.2), we shall introduce two different time scales, an interaction time  $t_{int}$  and the macro oscillation time  $t_{osc}$  which satisfy

$$t_{int} \ll t_{osc} \quad (2.17)$$

and

$$\begin{aligned} 0 &\approx |\omega_i - \omega_j| t_{int} \ll \omega_k t_{int}, \\ 0 &\approx |\omega_i - \omega_j| t_{int} \ll |\omega_i - \omega_j| t_{osc}, \end{aligned} \quad (2.18)$$

Since in the process of  $W$  boson decay (on-shell particles) discussed in this paper, the neutrino energies  $\omega_i$  are in the GeV range which is more than 10 orders of magnitude bigger than their differences, the above conditions can be easily satisfied. Equations (2.15) and (2.16) can be simplified as

$$\{a_{\mathbf{k},\sigma}^r(0), a_{\mathbf{k},\delta}^{r\dagger}(t \leq t_{int})\} = e^{i\omega t} \sum_l U_{\sigma,l} U_{\delta,l}^* \rho_{i,l}^{\mathbf{k}} \rho_{j,l}^{\mathbf{k}} + e^{-i\omega t} \sum_l U_{\sigma,l} U_{\delta,l}^* \lambda_{i,l}^{\mathbf{k}} \lambda_{j,l}^{\mathbf{k}}, \quad (2.19)$$

$$\{a_{\mathbf{k},\sigma}^r(0), b_{-\mathbf{k},\delta}^r(t \leq t_{int})\} = e^{i\omega t} \sum_l U_{\sigma,l} U_{\delta,l}^* (-i \rho_{i,l}^{\mathbf{k}} \lambda_{j,l}^{\mathbf{k}}) + e^{-i\omega t} \sum_l U_{\sigma,l} U_{\delta,l}^* i \lambda_{i,l}^{\mathbf{k}} \rho_{j,l}^{\mathbf{k}}, \quad (2.20)$$

For different flavors the above anticommutations are nonzero due to the dependence of  $\rho^{\mathbf{k}}$  and  $\lambda^{\mathbf{k}}$  on the  $m_i$ . And the interaction time scale  $t_{int}$  in them is instant comparing with the macro oscillation time scale  $t_{osc}$ , thus this effect is well separated from the oscillation. One of the consequences of this model is an exact neutrino oscillation formula obtained, e.g., for two-neutrino case the survival probability [15] is

$$P(\nu_e \rightarrow \nu_e) = 1 - \sin^2 2\theta \{ |U_k|^2 \sin^2[\Phi^+(t)] + |V_k|^2 \sin^2[\Phi^-(t)] \}, \quad (2.21)$$

here  $\Phi^+(t)$  and  $\Phi^-(t)$  are oscillation phases induced by positive and negative frequency parts;  $|V_k| = \sqrt{1 - |U_k|^2}$  with

$$|U_{\mathbf{k}}|^2 = \frac{1}{2} \sum_{r,s} |u_2^+(\mathbf{k}, r) u_1(\mathbf{k}, s)|^2 = 1 - O\left(\frac{m_i^2}{k^2}\right). \quad (2.22)$$

When  $|U_{\mathbf{k}}|^2 = 1$ , this exact probability becomes the usual Pontecovo formula. Corrections by *inequivalent vacua model* are at the order of  $O\left(\frac{m_i^2}{k^2}\right)$ .

### 3. Problems of neutrino weak states

Now we want to use the weak states defined above to derive the amplitudes of weak interaction processes described by charge current ( $CC$ ) and neutral current ( $NC$ ) in *Standard Model (SM)* of elementary particle physics, we get some ridiculous results after our calculations, such as negative energy neutrinos and flavor changing currents.

### 3.1 Negative energy neutrinos

Considering neutrinos produced in  $CC$  process, such as

$$W^+ \rightarrow e^+ + \nu_e, \quad (3.1)$$

the Hamiltonian responsible for this production vertex is

$$\mathcal{H} = -\frac{g}{\sqrt{2}}W_\mu^+(x)J_W^{\mu+} \equiv -\frac{g}{2\sqrt{2}}W_\mu^+(x)\bar{\nu}_e(x)\gamma^\mu(1-\gamma^5)e(x). \quad (3.2)$$

Assuming this process taking place at  $t = 0$ , the flavor vacuum at  $t = 0$  is defined as  $|0\rangle_f \equiv |0(t=0)\rangle_f$ ; then one  $e$ -neutrino state is  $|\nu_e(\mathbf{k}, r)\rangle \equiv a_{\mathbf{k},e}^{r\dagger}(0)|0\rangle_f$ ; and the Hermitian conjugation of this state is  $\langle\nu_e(\mathbf{k}, r)| \equiv {}_f\langle 0|a_{\mathbf{k},e}^r(0)$ . So the amplitude at tree level is expressed as

$$i\mathcal{M} = \langle\nu_e(\mathbf{k}, r)e^+(\mathbf{k}_e, r_e)|\{-i\int d^4x\mathcal{H}(x)\}|W^+(\mathbf{k}_W, \epsilon_\mu)\rangle. \quad (3.3)$$

Because  $e(x)$  and  $W_\mu^+(x)$  are both fields with definite mass quanta, their matrix elements can be derived easily as usual

$$\langle 0|W_\mu^+(x)|W^+(\mathbf{k}_W, \lambda)\rangle \propto \epsilon_\mu(\mathbf{k}_W, \lambda)e^{-i\omega_W t + i\mathbf{k}_W \cdot \mathbf{x}}, \quad (3.4)$$

$$\langle e^+(\mathbf{k}_e, r_e)|e(x)|0\rangle \propto v_e(\mathbf{k}_e, r_e)e^{i\omega_e t - i\mathbf{k}_e \cdot \mathbf{x}}. \quad (3.5)$$

We omit trivial constants in above expressions for simplicity, which have no influence on our results.  $\epsilon_\mu(\mathbf{k}_W, \lambda)$  is the polarization vector of the  $W^+$  boson, and  $v_e(\mathbf{k}_e, r_e)$  is the spinor of positron  $e^+$ .

Subtle differences come from neutrino sector. According to the *inequivalent vacua model*, we must use the flavor states to compute the matrix elements. Using (2.7), we can derive that

$$\begin{aligned} i\mathcal{M} \propto ig\delta^{(3)}(\mathbf{k}_W - \mathbf{k}_e - \mathbf{k}) \int dt & \\ \{ & {}_f\langle 0|a_{\mathbf{k},e}^r(0)a_{\mathbf{k},e}^{r\dagger}(t)|0\rangle_f \bar{u}_1(\mathbf{k}, r)\gamma^\mu(1-\gamma^5)v_e(\mathbf{k}_e, r_e) + \\ & {}_f\langle 0|a_{\mathbf{k},e}^r(0)b_{-\mathbf{k},e}^r(t)|0\rangle_f \bar{v}_1(-\mathbf{k}, r)\gamma^\mu(1-\gamma^5)v_e(\mathbf{k}_e, r_e) \} \\ & \epsilon_\mu(\mathbf{k}_W, \lambda)e^{i\omega_e t}e^{-i\omega_W t}. \end{aligned} \quad (3.6)$$

The flavor vacuum  $|0\rangle_f$  is defined at  $t = 0$ , so matrix elements in (3.6) can be expressed as

$${}_f\langle 0|a_{\mathbf{k},e}^r(0)a_{\mathbf{k},e}^{r\dagger}(t)|0\rangle_f = \{a_{\mathbf{k},e}^r(0), a_{\mathbf{k},e}^{r\dagger}(t)\}, \quad (3.7)$$

$${}_f\langle 0|a_{\mathbf{k},e}^r(0)b_{-\mathbf{k},e}^r(t)|0\rangle_f = \{a_{\mathbf{k},e}^r(0), b_{-\mathbf{k},e}^r(t)\}. \quad (3.8)$$

Now by using the expressions (2.15) and (2.16), we can get the final result of this amplitude

$$\begin{aligned} i\mathcal{M} \propto ig\delta^{(3)}(\mathbf{k}_W - \mathbf{k}_e - \mathbf{k}) \sum_i |U_{e,i}|^2 & \\ \{ & [\rho_{1,i}^{\mathbf{k}}]^2 \delta(\omega_W - \omega_e - \omega_i) + \lambda_{1,i}^{\mathbf{k}}]^2 \delta(\omega_W - \omega_e + \omega_i)] \\ & \bar{u}_1(\mathbf{k}, r)\gamma^\mu(1-\gamma^5)v_e(\mathbf{k}_e, r_e)\epsilon_\mu(\mathbf{k}_W, \lambda) \} + \\ \{ & [-i\rho_{1,i}^{\mathbf{k}}\lambda_{1,i}^{\mathbf{k}}\delta(\omega_W - \omega_e - \omega_i) + i\lambda_{1,i}^{\mathbf{k}}\rho_{1,i}^{\mathbf{k}}\delta(\omega_W - \omega_e + \omega_i)] \\ & \bar{v}_1(-\mathbf{k}, r)\gamma^\mu(1-\gamma^5)v_e(\mathbf{k}_e, r_e)\epsilon_\mu(\mathbf{k}_W, \lambda) \} \}. \end{aligned} \quad (3.9)$$

Among four parts of this amplitude, each has one  $\delta$  function of the energies, but two of them are  $\delta(\omega_W - \omega_e + \omega_i)$ . If it is interpreted as the conservation of energy, then there is negative energy neutrino with  $E = -\omega_i$ . Or contrarily, if we think neutrinos always have positive energy, this process will violate the principle of energy conservation.

In the limit of massless neutrinos, three of the four terms in (3.9) are vanishing and leaving only the first, which is entirely the same as the standard expression in *SM*. But here terms with  $\delta(\omega_W - \omega_e + \omega_i)$  are non-vanishing due to the dependence of  $\rho^{\mathbf{k}}$ ,  $\lambda^{\mathbf{k}}$  and  $\delta$  functions on the index  $i$ .

Entirely degenerated mass spectrum with  $m_i = m$  can also resolve this problem. It indicates that  $\rho^{\mathbf{k}} = 1$ ,  $\lambda^{\mathbf{k}} = 0$  and  $\omega_i = \omega$ , so the amplitude can be simplified as

$$i\mathcal{M} \propto ig\delta^{(4)}(k_W - k_e - k)\bar{u}_m(\mathbf{k}, r)\gamma^\mu(1 - \gamma^5)v_e(\mathbf{k}_e, r_e)\epsilon_\mu(\mathbf{k}_W, \lambda), \quad (3.10)$$

where  $u_m(\mathbf{k}, r)$  is the solution of  $(\not{k} - m)u_m(\mathbf{k}, r) = 0$ ,  $\mathbf{k}$  and  $k^0 \equiv \omega = \sqrt{\mathbf{k}^2 + m^2}$  are the momentum vector and the energy of  $\nu_e$  respectively. In fact, in this case there is no mixing at all, neutrino weak eigenstates are also mass eigenstates. It is a generalization of the case of massless neutrinos. It is mass differences (not masses) that are the crucial points of this problem. However, neutrino oscillation experiments, e.g., solar and atmospheric neutrino oscillations have confirmed the mass differences between different neutrinos [1–4, 8, 24, 25], thus this problem cannot be neglected.

### 3.2 Appearance of flavor changing currents

In fact, inspired by (2.15) and (2.16), we know that anticommutations for different flavors can be nonzero. So there exist non-trivial flavor changing *CC* and *NC* matrix elements at tree level. For example, if we consider process

$$W^+ \rightarrow e^+ + \nu_\mu, \quad (3.11)$$

when we use the Hamiltonian responsible for the standard *CC* interactions in (3.2), we get the tree-level amplitude

$$i\mathcal{M} = \langle \nu_\mu(\mathbf{k}, r)e^+(\mathbf{k}_e, r_e) | \{-i \int d^4x \mathcal{H}(x)\} | W^+(\mathbf{k}_W, \epsilon_\mu) \rangle. \quad (3.12)$$

Non-vanishing amplitude comes from the neutrino sector again. Because anticommutations at different time such as (2.16) are not the standard canonical relations, we get this unexpected amplitude. The final form of the amplitude can be expressed as

$$\begin{aligned} i\mathcal{M} \propto ig\delta^{(3)}(\mathbf{k}_W - \mathbf{k}_e - \mathbf{k}) \sum_i U_{\mu,i} U_{e,i}^* & \\ \{ \{ [\rho_{2,i}^{\mathbf{k}} \rho_{1,i}^{\mathbf{k}} \delta(\omega_W - \omega_e - \omega_i) + \lambda_{2,i}^{\mathbf{k}} \lambda_{1,i}^{\mathbf{k}} \delta(\omega_W - \omega_e + \omega_i)] & \\ \bar{u}_1(\mathbf{k}, r)\gamma^\mu(1 - \gamma^5)v_e(\mathbf{k}_e, r_e)\epsilon_\mu(\mathbf{k}_W, \lambda) \} + & \\ \{ [-i\rho_{2,i}^{\mathbf{k}} \lambda_{1,i}^{\mathbf{k}} \delta(\omega_W - \omega_e - \omega_i) + i\lambda_{2,i}^{\mathbf{k}} \rho_{1,i}^{\mathbf{k}} \delta(\omega_W - \omega_e + \omega_i)] & \\ \bar{v}_1(-\mathbf{k}, r)\gamma^\mu(1 - \gamma^5)v_e(\mathbf{k}_e, r_e)\epsilon_\mu(\mathbf{k}_W, \lambda) \} \}. & \end{aligned} \quad (3.13)$$

In the case of entirely degenerated mass spectrum with  $\rho^{\mathbf{k}} = 1$  and  $\lambda^{\mathbf{k}} = 0$ , the total amplitude is vanishing due to the unitarity of the mixing matrix. But in general case, the values of  $\rho^{\mathbf{k}}$  and  $\lambda^{\mathbf{k}}$  are different with respect to  $i$ , so besides negative energy neutrino problem, we encounter another severe problem: there are flavor changing currents such as  $W^+ \rightarrow e^+ + \nu_\mu$  in this framework.

Now let us estimate the branching ratio of this off-diagonal mode (3.11) to the normal diagonal mode (3.1). In (3.13) all particles are considered as in plane waves, and there are  $\delta$  functions of energy inside the sum. For different mass eigenstates the  $\delta$  functions are different, thus they can't be taken out of the sum. Under this consideration the branching ratio will be completely different from that in  $SM$  with zero neutrino mass. However this phenomenon is a general effect for mixing neutrino. It is a physical limit which describes an averaged neutrino oscillation effect, which is put as an appendix at the end of this paper. For an usual weak process, it is finished in a limited space-time with relations (2.17) and (2.18). The energy uncertainty makes the  $\delta$  function to be replaced by a wave package profile of energy distribution (e.g., a sharp gaussian). Different profiles with respect to  $i$  entirely overlap thus we can factorize the  $\delta$  functions out of the sum. This can be realized in mathematical formulism by using anticommutations (2.19) and (2.20) in the amplitude. Because in the rest frame of the  $W^+$  boson, the momentum of neutrinos almost equals to  $m_W/2$  ( $m_W$  is the mass of  $W$  boson, approximately equals 80 GeV), which is much larger than the masses of neutrinos. We expand the non trivial  $\rho^{\mathbf{k}}$  and  $\lambda^{\mathbf{k}}$  to high orders:  $\rho^{\mathbf{k}} \sim 1 - O(\frac{m_i^2}{k^2})$ ,  $\lambda^{\mathbf{k}} \sim O(\frac{m_i}{k})$ , and only consider the leading term in the two amplitudes. The estimated branching ratio will be

$$R_{\nu_\mu/\nu_e} \equiv \frac{\Gamma(W^+ \rightarrow e^+ + \nu_\mu)}{\Gamma(W^+ \rightarrow e^+ + \nu_e)} \sim \frac{|\sum_i -i\rho_{2,i}^{\mathbf{k}}\lambda_{1,i}^{\mathbf{k}}U_{\mu,i}U_{e,i}^*|^2}{|\sum_i \rho_{2,i}^{\mathbf{k}}\rho_{1,i}^{\mathbf{k}}|U_{e,i}|^2}. \quad (3.14)$$

One can see  $R_{\nu_\mu/\nu_e} \sim O(\frac{m_i^2}{k^2})$  (the first term in (3.13) gives  $O(\frac{m_i^4}{k^4})$  contribution; for terms with  $\delta(\omega_W - \omega_e + \omega_i)$ , we can't find a proper momentum satisfying the equation of  $\omega_W - \omega_e + \omega_i = 0$  for on-shell particles, so we omit their contributions). This is a pure flavor changing current effect, because it appears in the interaction time scale  $t_{int}$  which is instant comparing with the oscillation time scale  $t_{osc}$ . It is small for relativistic neutrinos and vanishes when neutrino is massless/degenerated. But it is the same order of magnitude of corrections in the *inequivalent vacua model* to the usual Pontecovo's formulas such as in (2.21) and (2.22). When we go beyond the relativistic limit, the corrections will be large, and the flavor changing current effect is also considerable. This phenomenon contradicts to our starting Hamiltonian (3.2) which is diagonal for neutrinos and charge leptons.

It also happens in the  $\nu_\tau$  family. These off-diagonal decay modes mean that the definition of weak neutrino states from mixing fields quantization in the *inequivalent vacua model* cannot properly describe neutrino interactions. In fact, another definition of neutrino weak states is on the basis of neutrino interactions [26, 27]. In our usual knowledge, neutrino weak states are defined to interact with corresponding charge leptons diagonally at tree level, just as the Hamiltonian in (3.2). And so far, the flavors of neutrinos in experiment are also identified by the signal of corresponding charge leptons. So the emergence of off-diagonal  $CC$  interactions will spoil the basis of flavor neutrino identification.



The problems discussed above also emerge in  $NC$  interactions. Let us discuss the decay of  $Z^0$  boson at tree level  $Z^0 \rightarrow \bar{\nu}_\sigma + \nu_\rho$ . Modes with  $\sigma = \rho$  indicate the usual interactions. But similarly to  $CC$  interactions, modes with  $\sigma \neq \rho$  are also nonzero due to the use of the Fock space in the *inequivalent vacua model*. These flavor changing neutral currents are also forbidden in the standard theory. Here we want to emphasize that the problems that arise are not specific of the process we consider, but quite general for processes including flavor neutrinos.

- *Discussions:* In QFT, particles are excitations of the corresponding fields, but for weak eigenfields, which is the mixing of mass eigenfields, it is difficult to define the corresponding quanta. At a glance it looks like that the *inequivalent vacua model* has overcome this difficulty. However, the artificial expansions of the weak eigenfields make it difficult to define an unique Fock space. It is improper to describe the weak interactions, and inconsistent with the flavor neutrino definition in weak interactions. The appearance of flavor changing currents is essential in this model. Its origin is the anticommutations in (2.19-2.20). In the previous study in ref. [22, 23], it has been pointed out that "the Fock spaces of flavor neutrinos are ingenious mathematical constructs without physical relevance". Our analysis supports that the problems occurred are in fact quite general. In the specific process ( $W$  boson decay), one can have a physical picture to see the problem.

#### 4. Conclusions

Physicists want to give an unified description of neutrino oscillation and neutrino interaction in the framework of QFT. In the *inequivalent vacua model*[14–19], they think the importance of this topic is the *Bogliubov* transformation between the two vacua. In this paper, we compute weak interaction vertices using the Fock space proposed in their model. From a  $CC$  process  $W^+ \rightarrow e^+ + \nu_e$ , we learn that in the complicated expression of (3.9), if  $\delta$  functions about energy is explained as energy conservation, negative energy neutrinos emerge in the process, otherwise this process violates the principle of energy conservation. We also compute a flavor changing process  $W^+ \rightarrow e^+ + \nu_\mu$  at tree level and find there is flavor changing current. Estimated branching ratio of this mode to the standard  $W^+ \rightarrow e^+ + \nu_e$  channel is at the order of  $O(\frac{m_\nu^2}{k^2})$ , which is the same order of the correction to standard Pontecovo's theory by the *inequivalent vacua model*. Existence of flavor changing currents will spoil our usual concepts on the definition of neutrino weak states in neutrino interaction. Only in the special case of neutrino mass degeneracy (massless limit is a particular situation of this case), these problems can be resolved. But the fact of neutrino oscillations has excluded this case.

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## A. Oscillation effect in weak decay

If we use real plane waves for particles. It means the space-time for the process is infinity, thus one expects that neutrino oscillation effect will appear in the result. In this case the processes for (3.13) and (3.9) are both incoherent superpositions of neutrino mass eigenstate processes with different energy  $\delta$  functions. Under this situation, the oscillation effect is bigger enough to neglect the *inequivalent vacua model* effect. We omit terms with  $\lambda^{\mathbf{k}}$ , and take  $\rho^{\mathbf{k}} \simeq 1$ . We can also omit dependence of the spinor calculations on neutrino mass for relativistic case. But dependence of the  $\delta$  functions on neutrino mass  $m_i$  can not be neglected in any case. After above simplification, we can immediately estimate the ratio of the two processes

$$R_{\nu_\mu/\nu_e} \equiv \frac{\Gamma(W^+ \rightarrow e^+ + \nu_\mu)}{\Gamma(W^+ \rightarrow e^+ + \nu_e)} \simeq \frac{\sum_i |U_{\mu,i} U_{e,i}^*|^2}{\sum_i |U_{e,i}|^4}. \quad (\text{A.1})$$

This is exactly the averaged (over time) oscillation ratio of  $P(\nu_e \rightarrow \nu_\mu)$  to  $P(\nu_e \rightarrow \nu_e)$ :

$$\bar{P}(\nu_e \rightarrow \nu_\mu) = \sum_i |U_{\mu,i} U_{e,i}^*|^2 \quad (\text{A.2})$$

$$\bar{P}(\nu_e \rightarrow \nu_e) = \sum_i |U_{e,i}|^4 \quad (\text{A.3})$$

The sum of three decay width  $W^+ \rightarrow e^+ + \nu_e$ ,  $W^+ \rightarrow e^+ + \nu_\mu$  and  $W^+ \rightarrow e^+ + \nu_\tau$  equals the width of  $W^+ \rightarrow e^+ + \nu_e$  in *SM*. That is because of the relation of

$$\sum_i \{|U_{e,i}|^4 + |U_{\mu,i} U_{e,i}^*|^2 + |U_{\tau,i} U_{e,i}^*|^2\} = 1. \quad (\text{A.4})$$

So it doesn't add extra width to the total width of  $W^+$  decay.

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